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ASSESSMENT OF DIRTY PAPER CODING WITH DECOUPLING NETWORK FOR
MITIGATING PILOT CONTAMINATION IN MASSIVE MIMO SYSTEM

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ABSTRACT

In this paper, we explore the possibilities of using dirty paper coding with decoupling network to mitigate the effects of pilot contamination in massive MIMO technology. Pilot contamination is one of the challenges that 5G poses and it has been shown to impose some constraints on the capacity of the system. We present the performance analysis on dirty paper coding with decoupling network in massive MIMO system. Mutual coupling is characterized by the receiving mutual impedance method to formulate the decoupling network operating matrix and design. A typical channel model of standard massive MIMO is presented and system performance is evaluated when coupling effects existed before and after decoupling process. Results indicate that output voltages of decoupling network can effectively be removed off the coupling effect. Error performance and channel capacity results demonstrate the promising potentials of dirty paper coding with efficient decoupling scheme in massive MIMO technology.

Keywords: *Coupling matrix, decoupling scheme, dirty paper coding, pilot contamination, receiving mutual impedance.*

I. INTRODUCTION

Massive MIMO (multiple-input multiple-output) has been recognized as a promising innovation for taking care of the demand for high data rates for mobile networks sooner rather than later. The emergence of new services like Internet-of-Things (IoT), machine-to-machine (M2M), e-learning, e-business, and others, have paved the way for existing cellular network technologies to be outfitted with high-capacity administrations. Fifth generation (5G) is relied upon to provide new technology components such as massive MIMO, device-to-device communications, machine communications, etc. [1], in light of the few favourable circumstances it has over existing advancements including bi-directional bandwidth sharing, substantial information transfer, subscriber supervision tools, peak upload and download speeds, among others. Massive MIMO provides significant improvements in spectral efficiency, obstruction alleviation, data rates and robustness, [1, 2], with respect to customary MIMO. It can be implemented with less-costly, low-powered amplifiers [3].

In massive MIMO system, pilot signals which are utilized to assess the channels can be polluted as a consequence of reuse of non-orthogonal pilot signals in a multi-cell framework [4, 5]. This causes the inter-cell interference that is relative to the base station (BS) antennas, [6], which decreases the achievable rates (throughput per cell and mean throughput per terminal) [7] in the system [8] and influences range effectiveness.

The utilization of training sequence symbols is not attainable practically speaking, for multi-cell as a result of short channel coherence time because of versatility of user terminals (UTs). This causes the phenomenon known as Pilot Contamination, a major impairment in the performance of massive MIMO systems [9, 10 and 11]. It introduces a finite signal to interference ratio (SIR) to the network which in turn causes saturation effects, i.e. system throughput doesn't increase even with expansion of number of BS antennas. This interference puts a blur on the quality of channel estimates [12, 13]. As the quantity of base stations expand, the pilots do not vanish. Hardware impairments [14]- [18] due to in-band and out-of-bound mutilations that meddle with training signals likewise cause interference in massive MIMO systems.

The objective of this paper is to investigate the system performance of Dirty Paper Coding (DPC) with decoupling network scheme in massive MIMO system to alleviate the effects of pilot contamination. The mutual coupling is characterized by the receiving mutual impedance method (RMIM) and its effects between antenna elements are included in the massive MIMO system model. The error performance and the system capacity of the coupled and decoupled antenna elements at the receiving ends are evaluated using Dirty Paper coding technique. There have been several research works of late on dirty paper coding with the work in [19] where dirty paper coding and time division multiple access (TDMA) transmission schemes were compared for MIMO broadcast channels. It was shown that dirty paper coding technique provided an upper bound on the sum-rate gain over the TDMA scheme.

This paper systematically presents the measurement of receiving mutual impedance of monopole array to formulate the decoupling network operating matrix and design. For illustration, coupling matrix under coupled and compensated voltages is determined. To give a full picture of system performance, coupling matrix is included in the massive MIMO system model with dirty paper system model. Figures of merit for mobile terminals such as channel capacity and error performance are investigated, and measurement results indicate the promising possibilities of dirty paper coding with efficient decoupling scheme to alleviate the pilot contamination effects in massive MIMO technology.

The outline of this paper is as follows: Section II presents pilot contamination, dirty paper coding and the receiving mutual impedance to formulate the coupling effect and to design the inserted decoupling scheme. The wireless system model, channel and coupling matrices are described in Section III. Simulation results and discussions are made in Section IV. Finally, concluding remarks are given in Section V.

II. DIRTY PAPER CODING AND DESIGN OF THE INSERTED DECOUPLING SCHEME

Pilot Contamination and Dirty Paper Coding

Since pilot contamination has been shown to impose the capacity limit on the system [7], lessening of pilot contamination will greatly enhance system performance. A couple of approaches to curb his pollution have been proposed by authors in [20] - [29], yet these approaches generally bring more intricacy into the system. Dirty paper coding is one method proposed to relieve the impact of pilot contamination. This is a technique that allows a base station to efficiently transmit data to multiple users at the same time. It has been shown that dirty paper coding achieves the sum-rate capacity of the multiple-antenna broadcast channel [30, 31 – 33] and the DPC achievable region is the largest known achievable region for the multiple-antenna broadcast channel. It assumes knowledge of the interfering signals at the transmitter.

We consider a broadcast channel with T receivers, $G > 1$ transmit antennas, and $R \geq 1$ receive antennas at each receiver. Let $x \in C^{G \times 1}$ be the transmitted vector signal and let $H_t \in C^{R \times G}$ be the channel matrix of receiver t where $H_t(i, j)$ represents the channel gain from transmit antenna j to antenna i of receiver t . The circularly symmetric complex Gaussian noise at the receiver t is represented by $n_t \in C^{R \times 1}$ where $n_t \sim N(0, 1)$. Let $y_t \in C^{R \times 1}$ be the received signal at receiver t . The received signal is mathematically represented as

$$y_t = H_t x + n_t \quad t = 1, \dots, T \quad (1)$$

For the single antenna broadcast channel, sum rate capacity is achieved by transmitting to the user with the largest channel norm. However, this is not generally true for a multiple transmit antenna broadcast channel. For the multiple-antenna channel, sum-rate capacity is achieved by using dirty paper coding to simultaneously transmit several users [30, 31 – 33]. The expression for the sum-rate capacity of the MIMO broadcast channel in terms of the dirty paper rate region is rather complicated. However, in [31], the dirty paper rate region is shown to be equal to the capacity region of the dual MIMO multiple-access channel with sum power constraint P , where the dual uplink is formed by changing the transmitter into a G -antenna receiver and changing each receiver into an R -antenna transmitter. The received signal in the dual MAC is given by

$$y_{MAC} = \sum_{i=1}^T H_i^\dagger x_i + n \quad (2)$$

where H_i^\dagger is the channel of each transmitter and the noise is the same as in the downlink. Notice that the dual channel matrix is simply the conjugate transpose of the downlink channel of each user. Due to the MAC-BC duality, the sum-rate capacity of the MIMO BC is equal to the maximum sum-rate achievable on the dual uplink with sum power constraint P :

$$C_{BC}(H, P) = \max_{\{Q_i: Q_i \geq 0, \sum_{i=1}^T \text{Tr}(Q_i) \leq P\}} \log \left| I + \sum_{i=1}^T H_i^\dagger Q_i H_i \right| \quad (3)$$

where each of the matrices Q_i is an $R \times R$ positive semi-definite covariance matrix. The expression in (3) is the sum-rate capacity of the dual uplink subject to sum power constraint P .

Receiving Mutual Impedance Formulation for the Mutual Coupling Effect

Receiving mutual impedance method is used to characterize the mutual coupling effect between two receiving monopoles for the study. The two parallel monopoles which are operating at a frequency of 2.4 GHz are placed on a metallic ground plane and connected to a $Z_0 = 50\Omega$ load. The monopoles have a length of 30mm each, radius of 2mm and an element separation of 25mm (0.2λ at 2.4 GHz). For the measurement set up in figure 1, the transmitting antenna is a horn antenna working between 2 – 4 GHz, whereas a separation of 50mm is given between transmitting antenna and receiving monopole array. Considering the concealed impacts of the metallic ground, figure 2 shows the measured S-parameters $S_{21}^{(1)}$, $S_{21}^{(2)}$ and S_{11} utilizing the procedure in [34]. If γ is complex and represents the square root of the transmitted power, the respective terminal voltages can be calculated as [35]

$$V_{21}^{(1)} = S_{21}^{(1)} \gamma \sqrt{Z_0}, \quad V_{21}^{(2)} = S_{21}^{(2)} \gamma \sqrt{Z_0}$$

and
$$V_{11} = S_{11} \gamma \sqrt{Z_0} \quad (4)$$

The voltage and current relationships can be expressed as

$$V_{21}^{(1)} = V_{11} + Z_{12} I_t^{(2)} \quad (5)$$

and

$$Z_{12} = I_t^{(1)} = \frac{V_{21}^{(1)}}{Z_o}, I_t^{(2)} = \frac{V_{21}^{(2)}}{Z_o}, \text{ and } I_t = \frac{V_{11}}{Z_o} \quad (6)$$

After some mathematical deductions, the mutual coupling between array elements is expressed as

$$Z_{12} = \left(\frac{V_{11} - V_{21}^{(1)}}{V_{21}^{(2)}} \right) Z_o = \left(\frac{S_{11} - S_{21}^{(1)}}{S_{21}^{(2)}} \right) Z_o \quad (7)$$

Consider a receiving antenna with array of N elements, the relationship between the uncoupled voltages U_k ($k = 1, 2, \dots, N$) and the received coupled voltages V_k can be written in a matrix notation as [36]

$$\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{bmatrix} = \begin{bmatrix} 1 & -\frac{Z_{12}}{Z_L} & \dots & -\frac{Z_{1N}}{Z_L} \\ -\frac{Z_{21}}{Z_L} & 1 & \dots & -\frac{Z_{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{Z_{N1}}{Z_L} & -\frac{Z_{N2}}{Z_L} & \dots & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad (8)$$

where Z_t^{ki} represents the mutual impedance between the k th and the i th antenna elements and Z_L is the terminal impedance connected to the antennas.

Decoupling Network Operating Matrix and Design

The compensation network consists of a power-divider with unequal power-dividing ratio and two rat-race couplers. The network uses no active circuit elements to minimize extra circuit noise [37]. As shown in Fig. 3, the power divider has three transmission lines, each having characteristics impedance of $\sqrt{2}Z_o$, where Z_o is the system characteristics impedance, but unequal electrical lengths. The electrical length θ can be calculated as [38]

$$\theta = \cos^{-1} \left(\frac{1}{\beta} \right) \quad (9)$$

where β is the power dividing ratio expressed as

$$\beta = \frac{1}{|z_t^{12}|}$$

$$z_t^{12} = \frac{Z_t^{12}}{Z_L} \quad (10)$$

From (8) the matrix operation of the decoupling network is expressed as

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{Z_{12}}{Z_L} \\ -\frac{Z_{21}}{Z_L} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 - \left(\frac{Z_{12}}{Z_L} \right) V_2 \\ V_2 - \left(\frac{Z_{21}}{Z_L} \right) V_1 \end{bmatrix} \quad (11)$$

where V_1 and V_2 are the couple voltages and the inputs to the network from the monopole terminals and the output voltages are U_1 and U_2 also known as the compensation voltages. We fabricate the circuit by using the substrate FR4 with dielectric constant 4.8, operating at a frequency of 2.4 GHz as shown in Fig. 3. The measured insertion loss between input and output ports of the decoupling network are shown in Fig. 4.

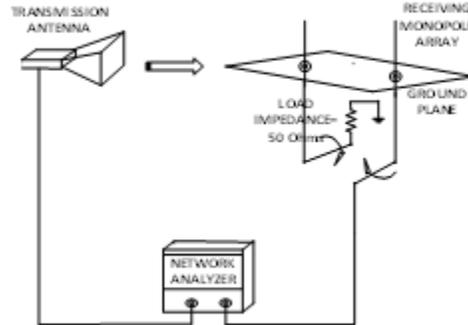


Fig. 1. Measurement of receiving mutual impedance of receiving monopole array in anechoic chamber

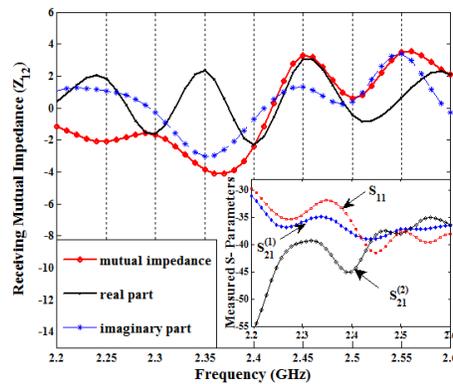


Fig. 2. Measured receiving mutual impedance of two quarter-wavelength side by side monopole receiving array (the inset shows measured S-parameters).

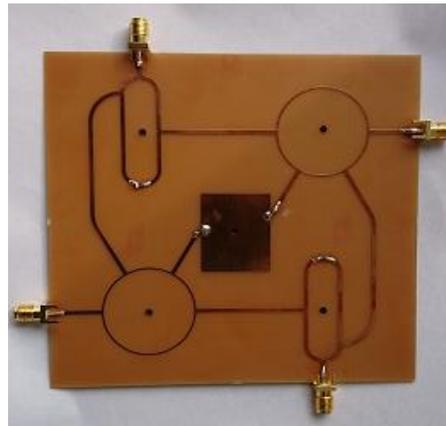


Fig.3. Photograph of the fabricated inserted decoupling network.

III. SYSTEM MODEL

A wireless system with typical channel model of standard massive MIMO is presented among users and base stations (BSs) [27]. In this paper, a $R(R \in \mathbb{Z})$ cell wireless system is considered and every cell comprises of a BS with A antennas and B users. The average transmit power of each BS antenna is represented by P_t and P_r indicating the average transmit power of user antenna. The BS of l -th ($0 < l \leq R$) cell is l -th

BS.

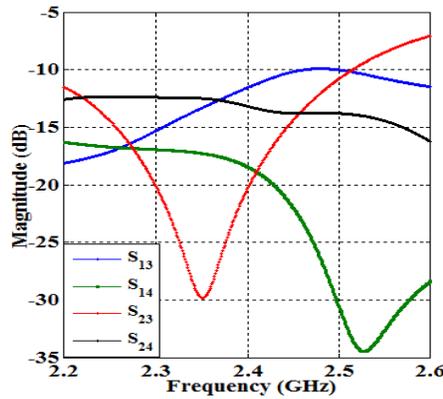


Fig.4. Measured insertion loss between input and output ports of the decoupling network.

The propagation factor between the l -th BS and the k -th user of the j -th ($0 < j \leq R$) cell is α_{ijk} , the propagation factor among the l -th and the j -th BS is α_{lj} . The channel vector between the l -th BS and the k -th user of the j -th cell is $h_{ljk} \in V^{M \times 1}$. Thus, in the l -th cell, the channel vector between the k -th user and the BS is h_{ljk} . The channels are considered to be frequency-flat and h_{ljk} remains constant during the coherence interval T ($T \gg \tau$). τ is the length of pilots. The channels are assumed of reciprocity, the channel factors are identical for both forward and backward link. If the channel matrix between the l -th BS and the j -th BS is $H_{lj} \in V^{M \times M}$, then $H_{jl} = H_{lj}^T$. The n -th row, m -th column of H_{lj} is H_{lj}^{nm} , representing the coefficient between the n -th antenna of the l -th BS and the m -th antenna of the j -th BS. Downlink communication is considered in this paper. The transmitter, typically the base station (BS), is highly elevated and not obstructed, with transmit antennas widely separated. Although the spatial correlation might exist at the transmitter because of the absence of scatterers at the surrounding environment, we just consider impacts of mutual coupling and channel correlation at the receiver where multiple antennas are placed closely to each other due to the relatively limited space. Hence, the channel H is given by [27]

$$H = ZR^{1/2}H_0 \quad (12)$$

where H_0 is the $B \times A$ channel matrix under independent, identically distributed (i.i.d) fading channel. Each entry of H_0 is a random variable with zero-mean and unity variance. Z and $R^{1/2}$ denote the mutual coupling and spatial correlation matrix at the receiver respectively. The expression for channel capacity can be written as the following

$$C(H) = \log_2 \det(I_N + \frac{\rho}{A} H_0^H R^{1/2H} Z^H ZR^{1/2} H_0) \quad (13)$$

$\rho = \text{tr}(E[x(t)x(t)^H]) / \sigma^2 = N / \sigma^2$. This is the average SNR per receive antenna. The received signal is

$$Y(t) = HX(t) + n(t) = ZR^{1/2}H_0 + n(t) \quad (14)$$

$n(t) = [n_1(t), n_2(t), \dots, n_B(t)]^T$ is the complex additive white Gaussian noise (AWGN) vector with zero-mean and variance $\sigma^2 / 2$ in each complex dimension. We consider the transmission and detection within one symbol period and as quasi-static fading is assumed, the time index t is omitted.

Error Performance

The generalized average error probability performance of M-PSK over fading channels is expressed as

$$\begin{aligned}
 k &= \int_0^\infty \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(-\frac{a^2\gamma}{2\sin^2\theta}\right) d\theta p_\gamma(\gamma) d\gamma \\
 &= \frac{1}{\pi} \int_0^{(M-1)\pi/M} \left[\int_0^\infty \exp\left(-\frac{a^2\gamma}{2\sin^2\theta}\right) p_\gamma(\gamma) d\gamma \right] d\theta \quad (15)
 \end{aligned}$$

where $a^2 = 2\sin^2 \pi / M$ after simple mathematical manipulations (15) becomes

$$k = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\gamma \left(-\frac{a^2}{2\sin^2\theta} \right) d\theta \quad (16)$$

$$k = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\gamma \left(-\frac{a^2}{2\sin^2\theta} \right) d\theta \quad (17)$$

In Rayleigh fading channel, error performance is expressed as

$$\begin{aligned}
 k \square k_r(a, \bar{\gamma}, M) &= \frac{1}{\pi} \int_0^{(M-1)\pi/M} \left(1 + \frac{a^2\bar{\gamma}}{2\sin^2\theta} \right)^{-1} d\theta \quad (18) \\
 &= \frac{M-1}{M} \left\{ 1 - \sqrt{\frac{a^2\bar{\gamma}/2}{1+a^2\bar{\gamma}/2}} \frac{M}{(M-1)\pi} \times \dots \right. \\
 &\quad \left. \dots \times \left[\frac{\pi}{2} + \tan^{-1} \left(\sqrt{\frac{a^2\bar{\gamma}/2}{1+a^2\bar{\gamma}/2}} \cot \frac{\pi}{M} \right) \right] \right\}
 \end{aligned}$$

where $\bar{\gamma}$ is the average signal to noise ratio.

IV. SIMULATION RESULTS AND DISCUSSIONS

To have a thorough understanding of the performance of dirty paper coding and decoupling network in massive MIMO system, coupling matrix under two different conditions are determined in an anechoic chamber using the set up in Fig.1, and incorporated into the system model for error performance analysis. In the first place, the scattering parameters on the coupled monopole array are measured to determine coupled voltages (V_1 and V_2) and coupling matrix.

This has been clarified in Section II. Secondly, the monopole antennas in the array are connected to the decoupling network through equivalent length coaxial links and scattering parameters of the output ports of the decoupling network are measured to determine the coupling matrix for the compensated voltages (U_1 and U_2). There are three different types of voltages listed in Table I. The last row in Table I is a ratio of the voltage obtained with monopole B to the voltage obtained with monopole A. It can be seen that the ratio of the compensated voltage is very close to the uncoupled voltages, demonstrating that the compensated voltages have successfully been removed of the coupling effect. On the other hand, the variations of cross correlation coefficients (ρ_{12}) recorded by the procedure in [39] under the two conditions demonstrated the lesser impacts of coupling on the compensated voltages.

Simulated results demonstrate the decoupling antennas with dirty paper coding have a significant improvement in the bit error performance and the channel capacity. Figure 5 shows the bit error performances of the coupled and compensated voltages for different frequencies. By observing fig. 5, the degradation in the performances of coupled voltages is evident to demonstrate the impact of higher mutual coupling. On the other hand, a close observation of

fig. 6 shows improved performances of the compensated voltages. It is a known fact that pilot contamination imposes capacity limit on the system, therefore, lessening the effect of pilot contamination will greatly enhance system performance. Our analysis demonstrates that efficient decoupling network with dirty paper coding technique will enhance system performance in massive MIMO.

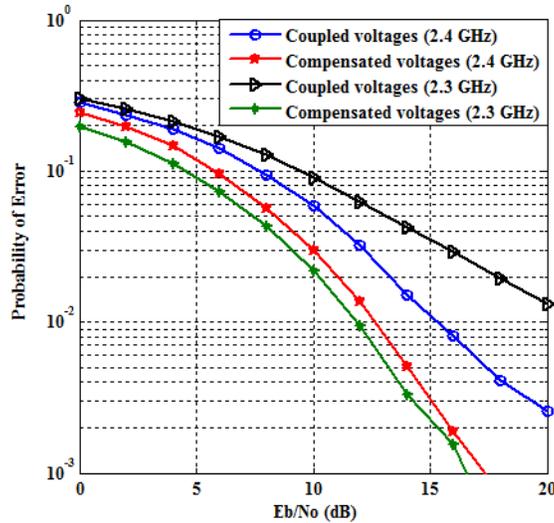


Fig. 5. Error performance for coupled and compensated voltages at different frequencies

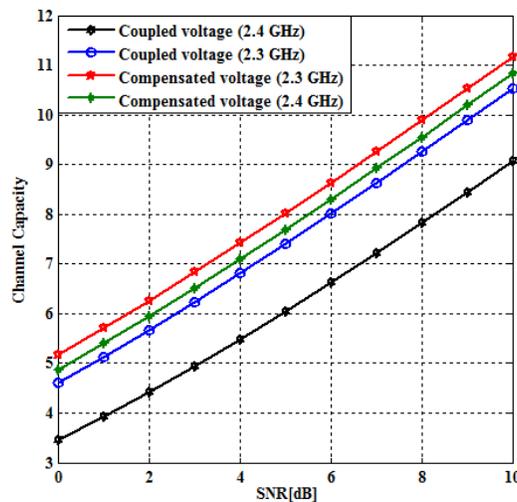


Fig. 6. Capacity per unit bandwidth for coupled and compensated voltages at different frequencies

V. CONCLUSION

The system performance of dirty paper coding with decoupling network in massive MIMO system is investigated in this paper. Mutual coupling between monopole arrays is characterized by the receiving mutual impedance method to formulate the decoupling network operating matrix and design.

A typical channel model of massive MIMO is presented and system performance is evaluated when coupling effects existed and after decoupling process. We showed results indicating that output voltages of decoupling network can effectively be removed off the coupling effect. Important figures of merit demonstrate the promising potentials of dirty paper coding with efficient decoupling scheme in massive MIMO system.

TABLE I

		Different Measured Voltages		
		Uncoupled Voltages (reference)	Coupled voltages	Compensated voltages
Monopole A	mag (mV)	16.64	12.4	11.55
	angle (°)	-160.64	-166.67	34.967
Monopole B	mag (mV)	16.54	15.42	12.30
	angle (°)	-139.56	-141.46	55.16
B/A	mag	0.9939	1.2199	1.065
	angle (°)	21.08	25.208	20.193

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